

## বিদ্যাসাগর বিশ্ববিদ্যালয় VIDYASAGAR UNIVERSITY

## **Question Paper**

## **B.Sc. General Examinations 2020**

(Under CBCS Pattern)

**Semester - I** 

**Subject: MATHEMATICS** 

Paper: DSC 1A/2A/3A-T (Differential Calculus)

Full Marks: 60
Time: 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any three from the following questions:

3×20

1. (a) If 
$$f(x) = \begin{cases} \frac{a \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 7, & \text{if } x = \frac{\pi}{2} \end{cases}$$
 for what values of 'a'  $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$ ?

(b) If 
$$y = x^{n-1} \log x$$
, then show that  $y_n = \frac{(n-1)!}{x}$ .

(c) If a and b are distinct real numbers, show that there exists a real number c between a and b such that  $a^2 + ab + b^2 = 3c^2$ .

(d) Find the radius of curvarture of the curvey  $y = xe^{-x}$  at the point where y is maximum.

(e) If 
$$f(x+y) = f(x).f(y)$$
,  $f'(0) = 3$ ,  $f(5) = 2$ , find  $f'(5)$ .  $4 \times 5 = 20$ 

- 2. (a) If  $f(x) = \tan x$ , then show that  $f''(0) n_{C_2} f^{n-2}(0) + n_{C_4} f^{n-4}(0) \dots = \sin \frac{n\pi}{2}$ 
  - (b) If u = f(x, y) and  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta}\right)^2$$
10+10

- 3. (a) If lx + my = 1 is a normal of the parabola  $y^2 = 4ax$ , then show that  $al^3 + 2alm^2 = m^2$ .
  - (b) Find the value of a, so that  $\lim_{x\to 0} \frac{a \sin x \sin 2x}{\tan^3 x}$  exists finitely and find the limit. 10+10
- 4. (a) A function f:[0,1] is continuous on [0,1]. Prove that there exists a point c in [0,1] such that f(c) = c.
  - (b) Using mean value theorem show that  $0 < \frac{1}{x} \log \left( \frac{e^x 1}{x} \right) < 1$ , for x > 0.
- 5. (a) If  $p_1$ ,  $p_2$  be the radii of curvarture at the extremities of any chord of the cardioid  $r = a(1 + \cos\theta)$ , which passes through the pole, then prove that  $p_1^2 + p_2^2 = \frac{16a^2}{9}$ .
  - (b) Find the infinite series expansion of the function  $\sin x, x \in \mathbb{R}$ . 10+10
- 6. (a) Show that  $f(x,y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$  has partical derivatives but is not continuous at (0,0).
  - (b) If the normal to the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  makes an angel  $\phi$  with the axis of x, show that its equation is  $y \cos \phi x \sin \phi = a \cos 2\phi$ . 10+10

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