বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

B.Sc. General Examination 2021

(CBCS)

1st Semester

MATHEMATICS

PAPER—DSC1AT / DSC2AT / DSC3AT

DIFFERENTIAL CALCULUS

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

THEORY : DSC1AT

Answer any *four* questions. 4×12

1. (a) Using Rolle's Theorem, find a point on the curve $y = \sin x + \cos x - 1,\ x \in \left[0, \frac{\pi}{2}\right]$, where the tangent is parallel to the x-axis.

(b) Find the n^{th} derivative of $\tan^{-1}t$. 6+6

2. (a) Let $f : R \to R$ be such that $f(x+y) = f(x) + f(y)$ for all x, y in R. Show that f(x) = ax, where x is an integer and f(1) = a.

(b) If y = a cos (log x) + b sin (log x), x > 0 then prove that (i) $x^2y_2 + xy_1 + y = 0$ and (ii) $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$. 5+7

3. (a) Determine whether the following function from R to R is differentiable and if differentiable find the derivative : $f(x) = 1 - |x - 1|$.

(b) Give an example of a function where it can be shown that the conditions of the Rolle's theorem are sufficient but not necessary.

(c) Find the Maclaurin's series for the function f(x) = sinx. 4+4+4

4. (a) State and proof the Taylor's theorem with Lagrange form of remainder.

(b) If a and b are distinct real numbers show that there exists a real number c between a and b such that $a^2 + ab + b^2 = 3c^2$.

(c) Determine the stationary point of $x^{\frac{1}{x}}$. 5+3+4

5. (a) Evaluate : $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{4}{5x}}$

(b) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x dy} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\left(1 - \frac{1}{2}\sin^2 u\right)\cos u}{\sin u}.$$ 5+7

6. (a) If V = $ax^2 + 2hxy + by^2$ then show that

$$\left(\frac{\partial V}{dx}\right)^2 \frac{\partial^2 V}{\partial y^2} - 2\frac{\partial V}{\partial x}\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial V}{\partial y}\frac{\partial^2 V}{\partial x^2} = 6(ab - h^2)V .$$

(b) If ρ_1 and ρ_2 be the radii of curvature at the end point P and D of conjugate diameters of the ellipse, prove that $\rho_1{}^{2/3} + \rho_2{}^{2/3} = \frac{\left(a^2 + b^2\right)}{(ab)^{2/3}}$.

6+6

7. (a) Verify Euler's Theorem when u(x, y) = $\frac{x(x^3 - y^3)}{x^3 + y^3}$.

(b) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $\frac{125}{16}$. 6+6

8. (a) If $x\cos\alpha + y\sin\alpha = p$ be the tangent of the curve $x^m y^n = \alpha^{m+n}$, then prove that $p^{m+n}m^n n^m = (m + n)^{m+n}\alpha^{m+n}\cos^m\alpha\sin^n\alpha$.

(b) Prove, $x - \frac{x^3}{6} < \sin x < x - \frac{x^3}{6} + \frac{x^5}{120}$, for all x > 0. 6+6

Answer any *six* questions. 6×2

9. Evaluate : $\lim\limits_{x \to \infty} \frac{1}{1 + n\sin^2 nx}$.

10. If f(x) = 2|x| + |x+2|, examine the existence of $f'(x)$ at x = 2.

11. If f(x) be differentiable at x = a, show that

$$\lim_{x \to a} \frac{(x+a)f(x) - 2af(a)}{x-a} = f(a) + 2af'(a)$$

12. Find y_n where $y = e^t \sin^2 t$.

13. Prove that sinx < x < tan x when $x \in \left(0, \frac{\pi}{2}\right)$.

14. Define essenetial discontinuity with an illustrated example.

15. If u = f(y − z, z − x, x − y) then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

16. Find the radius of curvature of the parabola $y^2 = 4ax$ at the vertex.

17. Find the equation of the line that is tangent to the graph of $y = \sqrt{x} - \frac{1}{\sqrt{x}} \, at \; x = 1$.

18. If |x| < 1, what is the coefficient of x^2 in the expression $\frac{\log_e(1+x)}{(1-x)^2}$.

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